An exact deformation analysis for the magneto-electro-elastic fiber-reinforced thin plate

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A B S T R A C T

A closed form expressions for bending problem of magneto-electro-elastic (MEE) rectangular thin plates are derived, the exact solutions for the deformation behaviors of the fiber-reinforced BaTiO₃/CoFe₂O₄ composites subjected to certain types of surface loads are analytically obtained. Based on Kirchhoff thin-plate theory, structural characteristics such as elastic displacements, electric potential and magnetic induction for magneto-electro-elastic (MEE) rectangular plates are investigated, the governing equation in terms of the transverse displacement is presented in a rather compact form due to the omission of the transverse shear deformation and rotatory inertia. The material coefficients for the MEE plate can be uniquely expressed by the volume-fraction (v.f.) of piezoelectric constituent BaTiO₃ in the fiber-reinforced composite, and are tabulated with 25% offset of the volume-fraction. The deformation variations of the MEE thin plate with closed-circuit electric restriction are evaluated analytically according to their specified boundary conditions, and the effects of the volume-fractions on the deformations variations are discussed. It can be found that all the results obtained by using the proposed model have reached good agreements with the other available research works, whereas, the present study provides a much simpler way in seeking the analytic solutions for the interactively coupled quantities of a multiphase medium.

1. Introduction

Smart materials such as piezoelectric and piezomagnetic materials have the properties that can be significantly changed in a controlled system by external stimuli, such as stress, electric or magnetic fields. Structure made of piezoelectric materials can produce a voltage when external stress is applied, nevertheless, in the reverse manner, a voltage across the structure will also produce stress within the structure. Applications of such piezoelectric structure include the production and detection of sounds and vibrations, generation of high voltages as well as electronic frequencies, and almost everyday facilities requiring device acting as a sensor or an actuator. In analog to piezoelectricity, the piezomagnetism is characterized by an interactive coupling between the system’s magnetic polarization and mechanical strain. In a piezomagnetic medium, one may induce a spontaneous magnetic moment by applying physical stress, or reversely obtain a physical deformation by applying a magnetic field.

Recently, the composites made of combinations between the piezoelectric and piezomagnetic components, either fiber-reinforced or layered, are generally referred as the magneto-electro-elastic (MEE) material. This kind of material possesses the ability to convert energy of magnetism, electricity or elasticity into another form and exhibits a specific magneto-electric...
effect which is not appeared in a single-phase piezoelectric or piezomagnetic material. Surprisingly, in some cases the magneto-electric effect of MEE composites can even be obtained two orders larger than that of a single-phase magneto-electric material with highest magneto-electro coefficient [1]. Due to these multiphase properties, the MEE material has been found increasing applications in making efficient smart and intelligent structures such as magnetic field probes, electric packing, acoustic, hydrophones, medical ultrasonic imaging and so on.

For the past few years, systematic investigations either in determining the material coefficient of such new type material or in analyzing the static or dynamic behavior under certain external conditions are vigorously proposed by many professional engineers and scientists. Li [2] studied the average magneto-electro-elastic fields in a multi-inclusion embedded in an infinite matrix and estimated the effective magneto-electro-elastic moduli of piezoelectric–piezomagnetic composites for both BaTiO3 fiber-reinforced CoFe2O4 and BaTiO3–CoFe2O4 laminate. It has been shown that the magneto-electro coupling demonstrated by magneto-electro coefficients vary with the volume faction of BaTiO3 and have opposite signs for fibrous and laminated composites, respectively. Wu and Huang [3] presented the closed-form solutions for the magnetoelectric coupling coefficients in fibrous composites with piezoelectric and piezomagnetic phases. In their investigation, the magnetoelectric coupling effect of the MEE intelligent composite is analytically determined based on the Mori-Tanaka theory. Aboudi [4] performed the micromechanical analysis for fully coupled electro-magneto-thermal-elastic composites by employing the homogenization micromechanical method and predicted the effective moduli of a fibrous composite with various value of volume fraction which represents the volume ratio between CoFe2O4 and BaTiO3. Pan [5] obtained the exact solution for three dimensional, anisotropic, linear magneto-elastic–elastic, simply-supported and multilayered rectangular plates under static loadings. It is stated that even for relatively thin plate, responses from an internal load are quite different to those from a surface load. Chen et al. [6] establish a micro-mechanical model to evaluate the effective properties of layered magneto-electro-elastic composites and the linear coupling effect between elasticity, electricity and magnetism of the MEE composite is accordingly derived. In their study, numerical results for a BaTiO3–CoFe2O4 composite with 2-2 connectivity are obtained, and the dependences of magneto-electro-elastic coefficients, the so-called product properties, of the composite on the volume fraction of BaTiO3 are clearly depicted. Wang and Shen [7] extended their previous works on piezoelectric media to study the general solution of three-dimensional problems in transversely isotropic magneto-electro-elastic media through five newly introduced potential functions. Chen and Lee [8] simplified the governing equations of the linear theory for the magneto-electro-thermal-elastic plate with transverse isotropy by introducing two displacement functions and stress functions. In their study, two new state space equations are established while selecting certain physical quantities as the basic unknowns. Wang et al. [9] performed a state vector formulation for the three dimensional, orthotropic and linearly magneto-electro-elastic multiple layered plate and expressed the basic unknowns by collecting not only the displacement, electric potential and magnetic potential but also some of the stresses, electric displacements, and magnetic induction. Pan and Heyliger [10] derived the analytical solutions in terms of the propagator matrices for the cylindrical bending of multilayered, linear, and anisotropic magneto-electro-elastic plates under simple-supported edge conditions. They considered the plate with a finite horizontal dimension in y-direction but infinite long in x-direction, and showed that the variations of the elastic, electric, and magnetic quantities along the thickness direction are strongly dependent upon the material property. A boundary integral formulation for the plane problem of magneto-electro-elastic media are performed by Ding and Jiang [11] using strict differential operator theory. They obtained the fundamental solutions for an infinite MEE plane in terms of four harmonic functions which satisfied a set of reduced second order partial differential equation for distinct eigenvalues case. Guan and He [12] derived the fundamental equation for the plane problem of a transversely isotropic magneto-electro-elastic media by applying the Almansi’s theorem and expressed all physical quantities by four harmonic functions for distinct and non-distinct cases.

In this study, a rather simple analytic solution for the deformations of the magneto-electro-elastic (MEE) rectangular plate under certain type of applied loads acting on the top or bottom surfaces are derived. By imposing the Kirchhoff thin-plate hypothesis on the plate constituent, the governing equation in terms of only the transverse displacement of the plate can be obtained and therefore a rather compact form indicating the multiple effects between elasticity, electricity and magnetism of the plate can be successfully presented. The MEE plate is chose to be made of the fiber-reinforced BaTiO3/CoFe2O4 lamina, which can be thought as a transversely isotropic magneto-electro-elastic medium and the material coefficients for such continuum can be expressed uniquely by introducing the volume-fraction of BaTiO3 in the multiphase composite. The corresponding deformation analysis regarding the elastic displacements, electric potential and magnetic induction of the MEE thin plate is evaluated through the formulation mentioned in this study. Some comparisons with previous literatures are made and great agreements are obtained which directly validate the proposed simplification for the MEE modeling.

2. Formulation

2.1. Basic equations

For a transversely isotropic magneto-electro-elastic medium with z-axis normal to the plane of isotropy, the constitutive equations in Cartesian coordinate system (x, y, z) can be written as [9]
2.2. General assumptions

The following assumptions are adopted [13]:

(1) The deflection of the midsurface is small compared with the thickness of the plate. The slope of the deflected surface is therefore very small and the square of the slope is a negligible quantity in comparison with unity.

(2) The midplane remains unstrained subsequent to bending.

(3) Plane sections initially normal to the midsurface remain plane and normal to that surface after the bending. This means that the vertical shear strains, $\gamma_{xz}$ and $\gamma_{yz}$, are negligible. The deflection of the plate is thus associated principally with bending strains. It is deduced therefore that the normal strain $\varepsilon_z$ resulting from transverse loading may also be omitted.

(4) The stress normal to the midplane, $\sigma_z$, is small compared with the other stress components and may be neglected. This supposition becomes unreliable in the vicinity of highly concentrated transverse loads.
As a consequence of the assumption (3), the strain-displacement relation reduce to

\[ e_x = \frac{\partial u}{\partial x}, \quad e_y = \frac{\partial v}{\partial y}, \quad 2\epsilon_{xy} = \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \]  

\[ e_z = \frac{\partial w}{\partial z} = 0. \quad 2\epsilon_{xz} = \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0. \quad 2\epsilon_{yz} = \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0. \]  

(7)

(8)

where \( \gamma_{ij} = \gamma_{ji} \) (i, j = x, y, z). Integrating Eq. (8), we can obtain

\[ w = w(x, y), \]  

indicating that the lateral deflection does not vary over the plate thickness. In a like manner, integration of expressions for \( \gamma_{xz} \) and \( \gamma_{yz} \) gives

\[ u = -z \frac{\partial w}{\partial x} + u_0(x, y), \quad v = -z \frac{\partial w}{\partial y} + v_0(x, y). \]  

(9)

Base on assumption (2) mentioned above, we conclude that \( u_0 = v_0 = 0 \). Thus

\[ u = -z \frac{\partial w}{\partial x}, \quad v = -z \frac{\partial w}{\partial y}. \]  

(10)

We see that Eq. (11) are consistent with assumption (3). Substitution of Eq. (11) into Eq. (7) yields

\[ e_x = -z \frac{\partial^2 w}{\partial x^2}, \quad e_y = -z \frac{\partial^2 w}{\partial y^2}, \quad \gamma_{xy} = -2z \frac{\partial^2 w}{\partial x \partial y}, \]  

(12)

the above formulas provide the strains at any point in the plate. As stated in the book written by Tzou [14], if the magneto-electro-elastic plate is thin, the transverse shear deformations and rotary inertias can be neglected. Also the transverse shear strains are negligible. In addition, the in-plane electric fields and magnetic fields are ignored, i.e. \( E_1 = E_2 = 0 \) and \( H_1 = H_2 = 0 \), only the transverse electric, \( E_3 \) and magnetic field, \( H_3 \), are considered in the present study. According to Maxwell theory, these two fields are related to the electric potential \( \phi \) and magnetic potential \( \psi \) by the following relations:

\[ E_x = \frac{\partial \phi}{\partial x} = 0, \quad E_y = -\frac{\partial \phi}{\partial y} = 0, \quad E_z = -\frac{\partial \phi}{\partial z}, \]  

(13)

\[ H_x = -\frac{\partial \psi}{\partial x} = 0, \quad H_y = -\frac{\partial \psi}{\partial y} = 0, \quad H_z = -\frac{\partial \psi}{\partial z}. \]  

(14)

### 2.3. Derivations

If a thin plate structure is considered, the following stress, moment and shear force are defined,

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy} \\
Q_x \\
Q_y
\end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy} \\
\tau_{xz} \\
\tau_{yz}
\end{bmatrix} dz, 
\]  

\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} ydz. 
\]  

(15)

(16)

After integrating Eqs. (15) and (16) with respect to the direction of plate thickness, \( dz \), neglecting the body force term, five balance equations about the plate bending problem can be derived. If no shear force on the surface, the balanced equation of a bending plate due to lateral load can be further simplified into

\[
\frac{\partial^2 M_x}{\partial y^2} + \frac{\partial^2 M_y}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial x \partial y} = -\Delta p(x, y), \]  

(17)

where \( \Delta p(x, y) \equiv p_{\text{upper}}(x, y) - p_{\text{lower}}(x, y) \) denotes the difference of applied loads between top surface and bottom surface of the plate. Substituting Eqs. (7), (8), (12)–(14) into the constitute Eqs. (1)–(3), one can obtain the reduced extended traction vectors [5] as stated below
\[ \sigma_x = c_{11} \left( -z \frac{\partial^2 w}{\partial x^2} \right) + c_{12} \left( -z \frac{\partial^2 w}{\partial y^2} \right) + e_{31} \frac{\partial \psi}{\partial z} + q_{31} \frac{\partial \phi}{\partial z}, \]
\[ \sigma_y = c_{12} \left( -z \frac{\partial^2 w}{\partial x^2} \right) + c_{11} \left( -z \frac{\partial^2 w}{\partial y^2} \right) + e_{31} \frac{\partial \phi}{\partial z} + q_{31} \frac{\partial \psi}{\partial z}, \]
\[ \tau_{xy} = c_{66} \left( -z \frac{\partial^2 w}{\partial x \partial y} \right) - 2 c_{66} z \frac{\partial^2 w}{\partial x \partial y}, \]
\[ D_z = e_{31} \frac{\partial u}{\partial x} + e_{32} \frac{\partial v}{\partial y} - e_{33} \frac{\partial \psi}{\partial z} - d_{33} \frac{\partial \phi}{\partial z}, \]
\[ B_z = q_{31} \frac{\partial u}{\partial x} + q_{32} \frac{\partial v}{\partial y} - d_{33} \frac{\partial \phi}{\partial z} - \mu_{33} \frac{\partial \psi}{\partial z}. \]

Substitute Eqs. (19) and (20) into Eqs. (5) and (6), we have the following two algebraic equations:
\[ e_{31} \frac{\partial}{\partial z} \left( -z \frac{\partial^2 w}{\partial x^2} \right) + e_{31} \frac{\partial}{\partial z} \left( -z \frac{\partial^2 w}{\partial y^2} \right) - e_{33} \frac{\partial^2 \phi}{\partial z^2} - d_{33} \frac{\partial^2 \psi}{\partial z^2} = 0, \]
\[ q_{31} \frac{\partial}{\partial z} \left( -z \frac{\partial^2 w}{\partial x^2} \right) + q_{31} \frac{\partial}{\partial z} \left( -z \frac{\partial^2 w}{\partial y^2} \right) - d_{33} \frac{\partial^2 \phi}{\partial z^2} - \mu_{33} \frac{\partial^2 \psi}{\partial z^2} = 0, \]

this implies
\[ -e_{31} \frac{\partial^2 w}{\partial x^2} - e_{31} \frac{\partial^2 w}{\partial y^2} - e_{33} \frac{\partial^2 \phi}{\partial z^2} - d_{33} \frac{\partial^2 \psi}{\partial z^2} = 0, \]
\[ -q_{31} \frac{\partial^2 w}{\partial x^2} - q_{31} \frac{\partial^2 w}{\partial y^2} - d_{33} \frac{\partial^2 \phi}{\partial z^2} - \mu_{33} \frac{\partial^2 \psi}{\partial z^2} = 0, \]

and it can be rewrite as
\[ e_{33} \frac{\partial^2 \phi}{\partial z^2} + d_{33} \frac{\partial^2 \psi}{\partial z^2} = e_{31} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \equiv -e_{31} \nabla^2 w, \]
\[ d_{33} \frac{\partial^2 \phi}{\partial z^2} + \mu_{33} \frac{\partial^2 \psi}{\partial z^2} = q_{31} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \equiv -q_{31} \nabla^2 w, \]

thus we can have
\[ \frac{\partial^2 \phi}{\partial z^2} = -\frac{A_1}{A} \nabla^2 w \quad \text{and} \quad \frac{\partial^2 \psi}{\partial z^2} = -\frac{A_2}{A} \nabla^2 w, \]

where
\[ A \equiv \det \begin{pmatrix} e_{33} & d_{33} \\ d_{33} & \mu_{33} \end{pmatrix} = e_{33} \mu_{33} - d_{33}^2, \]
\[ A_1 \equiv \det \begin{pmatrix} e_{31} & d_{33} \\ q_{31} & \mu_{33} \end{pmatrix} = e_{31} \mu_{33} - d_{33} q_{31}, \]
and
\[ A_2 \equiv \det \begin{pmatrix} \mu_{33} & e_{31} \\ d_{33} & q_{31} \end{pmatrix} = e_{33} q_{31} - d_{33} e_{31}. \]

It can be derived from Eq. (27) that
\[ \frac{\partial \phi}{\partial z} = -\frac{A_1}{A} z \nabla^2 w + \phi_1(x, y) \]

and
\[ \frac{\partial \psi}{\partial z} = -\frac{A_2}{A} z \nabla^2 w + \psi_1(x, y), \]

where \( \phi_1(x, y) \) and \( \psi_1(x, y) \) represent the variations of electric field and magnetic field in the thickness direction while the plate is vibrating.

Substitute Eqs. (31) and (32) into Eq. (18) and integrate them respect to \( z \)-direction as shown in Eqs. (15) and (16) will lead to the following results:
After thus the governing equation for a thin magneto-electro-elastic bending plate can be stated as follows:

\[
M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{x} z dz \\
= \int_{-\frac{h}{2}}^{\frac{h}{2}} c_{11} \left( -z \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial x^2} + e_{31} \left( -\frac{A_1}{A} \xi^2 w + \phi_1(x,y) \right) + q_{31} \left( -\frac{A_2}{A} \xi^2 w + \phi_1(x,y) \right) \right) dz \\
= -\frac{h^3}{12} \left[ c_{11} \frac{\partial^2 w}{\partial x^2} + c_{12} \frac{\partial^2 w}{\partial y^2} + e_{31} \frac{A_1}{A} \xi^2 w + q_{31} \frac{A_2}{A} \xi^2 w \right],
\]

(33)

\[
M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{y} z dz = -\frac{h^3}{12} \left[ c_{12} \frac{\partial^2 w}{\partial x^2} + c_{11} \frac{\partial^2 w}{\partial y^2} + e_{31} \frac{A_1}{A} \xi^2 w + q_{31} \frac{A_2}{A} \xi^2 w \right]
\]

(34)

and

\[
M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xy} z dz = \int_{-\frac{h}{2}}^{\frac{h}{2}} c_{66} \left( -2z \frac{\partial^2 w}{\partial x \partial y} \right) dz = -\frac{h^3}{12} \cdot 2c_{66} \frac{\partial^2 w}{\partial x \partial y}
\]

(35)

Substitute Eqs. (33) and (34) into Eq. (17) will result in the following expression for bending problem of a magneto-electro-elastic thin plate,

\[
\frac{h^3}{12} \left\{ \frac{\partial^2}{\partial x^2} \left[ c_{11} \frac{\partial^2 w}{\partial x^2} + c_{12} \frac{\partial^2 w}{\partial y^2} + e_{31} \frac{A_1}{A} \xi^2 w + q_{31} \frac{A_2}{A} \xi^2 w \right] \right. \\
\left. + 4c_{66} \frac{\partial^2}{\partial x \partial y} \left( \frac{\partial^2 w}{\partial x^2} + c_{11} \frac{\partial^2 w}{\partial y^2} + e_{31} \frac{A_1}{A} \xi^2 w + q_{31} \frac{A_2}{A} \xi^2 w \right) \right\} = Ap(x,y)
\]

(36)

Since for transversely isotropic material, the relation \( c_{11} = c_{12} + 2c_{66} \) holds, the above equation can then be further simplified into

\[
\frac{h^3}{12} \left\{ c_{11} \left( \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + e_{31} \frac{A_1}{A} \xi^4 w + q_{31} \frac{A_2}{A} \xi^4 w \right\} = Ap(x,y),
\]

(37)

thus the governing equation for a thin magneto-electro-elastic bending plate can be stated as follows:

\[
\left\{ D\frac{\partial^4 w}{\partial x^4} + E\frac{\partial^4 w}{\partial y^4} + M\frac{\partial^4 w}{\partial y^4} \right\} = Ap(x,y),
\]

(38)

where \( D \equiv c_{11} h^3, \quad E \equiv c_{11} h^3, \quad \frac{1}{4}, \quad M \equiv 2c_{66} h^3, \quad \frac{1}{4}, \) represents the plate rigidity, effective rigidities due to the presence of electricity and magnetism, respectively. It should be noted that Eq. (38) represents a rather compact form for the deformation of a MEE thin plate based on the fact that the proposed approach is much simpler than the others in the existing literatures, and can only be applied to the plate with length-to-thickness ratio greater than 10.

2.4. Deformations due to applied load

In seeking for the solution to Eq. (38), we can assume the following expression for the transverse deflection of the MEE plate,

\[
w(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}X_m(x)Y_n(y),
\]

(39)

where \( X_m(x) \) and \( Y_n(y) \) are the homogeneous solutions of Eq. (38) and can be determined according to the specified boundary conditions. Some commonly seen mode shapes and the corresponding eigenvalues with respect to various boundary conditions are tabulated in Table 1 as a reference. It should be noted that not only \( X_m(x) \) and \( Y_n(y) \) satisfy the corresponding boundary conditions but also possess the important orthogonality, i.e., we will have

\[
\int_0^{L_x} X_m(x)X_M(x)dx = \begin{cases} ||X_m(x)||, & \text{if } m = M \\ 0, & \text{if } m \neq M \end{cases} \equiv ||X_m(x)||\delta_{mm}
\]

(40)

and

\[
\int_0^{L_y} Y_n(y)Y_N(y)dy = \begin{cases} ||Y_n(y)||, & \text{if } n = N \\ 0, & \text{if } n \neq N \end{cases} \equiv ||Y_n(y)||\delta_{nn}.
\]

(41)

After \( X_m(x) \) and \( Y_n(y) \) are determined, we can further expand the applied load on the top or bottom surface of the plate into the generalized double Fourier series as
plate, there are normally two kinds of cases to be discussed. According to Eq. (11),
the exact solution for the mechanical displacements along the transverse direction is

Thus we have the exact solution for the transverse deflection due to applied load
\( \frac{d^2w}{dx^2} + \frac{d^2w}{dy^2} + E \frac{d^2w}{dt^2} = 0 \)

By substituting Eqs. (39) and (42) into Eq. (38) and collecting the constant term, we can have the following equation:

\[
X_m(x) = \sin \left( \frac{m\pi x}{L} \right)
\]

where
\( m = 1, 2, 3, \ldots \)

and
\( \gamma = 1.000777 \)

\( \gamma_2 = 1.000001 \)

\( \gamma_3 = 1.000000 \)

Fixed–Free

\[
X_m(x) = \cos \left( \frac{m\pi x}{L} \right)
\]

where
\( \gamma_1 = 0.734096 \)

\( \gamma_2 = 1.018466 \)

\( \gamma_3 = 0.9999225 \)

Fixed–fixed

\[
X_m(x) = \cos \left( \frac{m\pi x}{L} \right) - \gamma_m \sin \left( \frac{m\pi x}{L} \right)
\]

where
\( \gamma_1 = 0.982502 \)

\( \gamma_2 = 1.000777 \)

\( \gamma_3 = 0.999966 \)

Free–free

\[
X_m(x) = \cos \left( \frac{m\pi x}{L} \right) - \gamma_m \sin \left( \frac{m\pi x}{L} \right)
\]

where
\( \gamma_1 = 0.982502 \)

\( \gamma_2 = 1.000777 \)

\( \gamma_3 = 0.999966 \)

In which the Fourier coefficient \( p_{mn} \) can be determined as follows

\[
p_{mn} = \frac{1}{\|X_m(x)\| \|Y_n(y)\|} \int_0^L \int_0^L p(x, y) X_m(x) Y_n(y) dx dy.
\]

By substituting Eqs. (39) and (42) into Eq. (38) and collecting the constant term, we can have the following equation:

\[
(D + E + M)\left( x_m^2 + 2x_m^2 \beta_n^2 + \beta_n^4 \right) A_{nm} = p_{mn},
\]

where \( x_m \) and \( \beta_m \) are the eigenvalues corresponding to the specific boundary conditions. Furthermore, the magnitude of the transverse deflection, \( A_{nm} \), can be determined, i.e.,

\[
A_{nm} = \frac{p_{mn}}{(D + E + M)\left( x_m^2 + 2x_m^2 \beta_n^2 + \beta_n^4 \right)}.
\]

Thus we have the exact solution for the transverse deflection due to applied load

\[
w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{p_{mn}}{(D + E + M)\left( x_m^2 + 2x_m^2 \beta_n^2 + \beta_n^4 \right)} X_m(x) Y_n(y),
\]

and according to Eq. (11), the exact solution for the mechanical displacements along \( x- \) and \( y- \) directions can be expressed as

\[
u = -\frac{\partial w}{\partial y} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{-p_{mn}}{(D + E + M)\left( x_m^2 + 2x_m^2 \beta_n^2 + \beta_n^4 \right)} 2X_m(x) Y_n(y),
\]

As for the electric potential and magnetic potential as

\[
\phi(x, y, z) = \left\{ \begin{array}{ll}
0 & \text{if } z > h/2 \\
0 & \text{if } z < -h/2
\end{array} \right.
\]

By carrying out the anti-derivatives for Eqs. (31) and (32), we can have the following expressions for electric potential and magnetic potential as
Examining Eqs. (57)–(60), it can be found that these four equations can neither provide any information about determining
therefore, the open-circuit conditions will lead to
Therefore, by solving the above two equations, one can get
thus the exact solution for the electric potential due to applied load can be expressed as
By the same token, we can have the following exact solution for the magnetic potential due to applied load
Case II: Open-circuit, i.e., $D_z(x, y, \pm h/2) = 0$ and $B_z(x, y, \pm h/2) = 0$
Substituting Eqs. (11), (31) and (32) into Eqs. (19) and (20), the solutions for the electric displacement and magnetic
induction in thickness direction can be expressed as
therefore, the open-circuit conditions will lead to
and
Exchanging Eqs. (57)–(60), it can be found that these four equations can neither provide any information about determining
the expressions for $\phi_1(x, y)$ and $\psi_1(x, y)$, nor for the terms $\phi_0(x, y)$ and $\psi_0(x, y)$. This is because the variations of electric
displacement and magnetic displacement along thickness direction are vanished under the proposed methodology, i.e., $\frac{\partial \phi}{\partial z} = \frac{\partial \psi}{\partial z} = 0$, therefore it is expected to see the linear dependency of these two quantities on the $z$ variable. In that sense, open-circuit boundary will result in a trivial solution with the displacement components approaching to zero and makes the expressions of $\phi_1(x, y)$ and $\psi_1(x, y)$ being negligible as can be detected from Eqs. (55) and (56). As a result, two more boundary conditions will be needed in order to resolve the expressions for the terms $\phi_0(x, y)$ and $\psi_0(x, y)$. For that reason, in this study, we do not specifically focus on the open-circuit case and only adopt the closed-circuit boundary condition as our numerical examples discussed in the next section.

It should be noted that the in-plane electric fields and magnetic fields can be ignored if the plate thickness is very small (e.g. $H < 10L_z$), and only the transverse electric field, $E_z$, and magnetic field, $H_z$, are related to the electric potential $\phi$ and magnetic potential $\psi$ in the follow form according to the Maxwell’s equations, i.e., Eqs. (13) and (14).

$$E_z = \frac{\partial \phi}{\partial z} = \frac{A_1}{\Delta} z^2 \nabla^2 w(x, y) - \phi_1(x, y), \quad (61)$$

$$H_z = -\frac{\partial \psi}{\partial z} = \frac{A_2}{\Delta} z^2 \nabla^2 w - \psi_1(x, y) \quad (62)$$
in which the terms \( \phi_1(x, y) \) and \( \psi_1(x, y) \) can be determined by considering the closed-circuit case on the plate surfaces. Also the exact solutions for the electric displacement and magnetic induction in thickness direction can be expressed as

\[
D_z = \left( -\varepsilon_{31} + \varepsilon_{33} \frac{A_1}{A} + d_{33} \frac{A_2}{A} \right) z \nabla^2 w - \varepsilon_{33} \phi_1(x, y) - d_{33} \psi_1(x, y),
\]

\[
B_z = \left( -q_{31} + d_{33} \frac{A_1}{A} + \mu_{33} \frac{A_2}{A} \right) z \nabla^2 w - d_{33} \phi_1(x, y) - \mu_{33} \psi_1(x, y).
\]

(63) (64)

And the stress distributions are respectively

\[
\sigma_x(x, y, z) = -c_{11} A_{mn} z X_m'(x) Y_n(y) - c_{12} A_{mn} z X_m'(x) Y_n'(y) - \left( \varepsilon_{31} \frac{A_1}{A} + q_{31} \frac{A_2}{A} \right) A_{mn} z (X_m''(x) Y_n(y) + X_m(x) Y_n''(y)) + e_{31} \phi_1(x, y) + q_{31} \psi_1(x, y),
\]

\[
\sigma_y(x, y, z) = -c_{12} A_{mn} z X_m'(x) Y_n(y) - c_{11} A_{mn} z X_m'(x) Y_n'(y) - \left( e_{31} \frac{A_1}{A} + e_{32} \frac{A_2}{A} \right) A_{mn} z (X_m''(x) Y_n(y) + X_m(x) Y_n''(y)) + e_{31} \phi_1(x, y) + q_{31} \psi_1(x, y),
\]

\[
\tau_{xy} = -2c_{56} A_{mn} z X_m'(x) Y_n'(y).
\]

The above equations provide the analytic solutions to the physical quantities of a magneto-electro-elastic thin plate subjected to surface applied load and can be decided according to the transverse deflection \( w(x, y) \) and potential parameters \( \phi_1(x, y, z) \) and \( \psi_1(x, y, z) \).

3. Numerical examples and discussions

In this section, some examples based on the proposed model for the magneto-electro-elastic rectangular thin plate subjected to external loads and closed-circuit electric restrictions are presented. For the first a few examples, the author demonstrates some special cases which have been verified in previous research works in order to validate the present study and tell the differences with other available literatures.

**Example 1.** The first example illustrated here is a simply-supported square plate made of purely piezoelectric material, PZT-5H, which is subjected to a uniform applied load on the top and bottom surfaces, i.e., \( \Delta p(x, y) = P_0 \). The material constant for PZT-5H can be found in the paper written by Bisegna and Maceri [15], and is tabulated in Table 2 in the present study. The thickness-to-side ratio of the plate is set to be 1/20 in order to match the requirement for thin-plate theory and the data obtained here are compared with the case of “thin-plate limit” as remarked in the study made by Bisegna and Maceri [15]. Fig. 1(a)–(d) shows the variations of in-plane displacement \( u_i(L_x/L_y, L_y/2, z) \), central transverse deflection \( w(L_x/2, L_y/2, z) \), central electric potential \( \phi(L_x/2, L_y/2, z) \) and central in-plane stress \( \sigma_{11}(L_x/2, L_y/2, z) \) along thickness direction at the specified location. In order to compare, the following dimensionless formula for the related functions are implemented.

| Material constant for some single-phase continuums and fiber-reinforced BaTiO\(_3\)/CoFe\(_2\)O\(_4\) MEE multiphase composites, partially cited from Buchanan [16]. |

<table>
<thead>
<tr>
<th>Material constant</th>
<th>PZT-5H</th>
<th>CoFe(_2)O(_4)</th>
<th>25% Lamina</th>
<th>50% Lamina</th>
<th>75% Lamina</th>
<th>BaTiO(_3)</th>
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<tr>
<td>( c_{11} )</td>
<td>126</td>
<td>286</td>
<td>245</td>
<td>215</td>
<td>186</td>
<td>166</td>
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<td>( c_{12} )</td>
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<td>173</td>
<td>144</td>
<td>112</td>
<td>90</td>
<td>77</td>
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<tr>
<td>( c_{13} )</td>
<td>84.1</td>
<td>170</td>
<td>144</td>
<td>112</td>
<td>90</td>
<td>78</td>
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<tr>
<td>( c_{24} )</td>
<td>117</td>
<td>269.5</td>
<td>235</td>
<td>210</td>
<td>181</td>
<td>162</td>
</tr>
<tr>
<td>( c_{44} )</td>
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<td>45.3</td>
<td>46</td>
<td>50</td>
<td>51</td>
<td>43</td>
</tr>
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<td>( e_{31} )</td>
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<td>0</td>
<td>−1.5</td>
<td>−2.8</td>
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<tr>
<td>( e_{33} )</td>
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<td>4.2</td>
<td>8.7</td>
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<td>0.3</td>
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<td>0.1</td>
<td>0.25</td>
<td>0.5</td>
<td>11.2</td>
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<td>−2.00</td>
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<tr>
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<td>0</td>
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<td>2350</td>
<td>2750</td>
<td>1800</td>
<td>0</td>
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\( (c_{ij} \text{ in } \text{N/m}^2, e_{ij} \text{ in } \text{C/m}^2, q_{ij} \text{ in } \text{N/Am}^2, e_{ij} \text{ in } 10^{-9} \text{C}^2/\text{N m}^2, \text{ and } \mu_{ij} \text{ in } 10^{-9} \text{Ns}^2/\text{C}^2 \text{ and } m_{ij} \text{ in } 10^{-11} \text{Ns/VC}). \)
\[
s_1(z) := \frac{\pi^3 H^2 C}{3 A^1 P_0} u_x(L_x, L_y/2, H \zeta), \quad s_2(z) := \frac{\pi^4 H^2 C}{3 A^4 P_0} w(L_x/2, L_y/2, H \zeta),
\]

\[
\phi(z) := \frac{2 \pi^2 H x C}{A^2 F P_0} \phi(L_x/2, L_y/2, H \zeta), \quad T_1(z) := \frac{\pi^2 H^2}{6(1 - \zeta) A^2 P_0} \sigma_{11}(L_x/2, L_y/2, H \zeta),
\]

where \(z := z/H\), \(C := 96.2 \times 10^6\), \(A := Lx\), \(\varepsilon := 27.6 \times 10^{-6}\), \(F := 36.3\) and \(\zeta := C_{66}/C\) are defined in the referenced paper. As we can observe from these figures, they are all having the same trends with the corresponding quantities in Bisegna’s paper, except that the values are not exactly the same. This is because that the collecting terms for the summation in this paper is 10 instead of 100 in Bisegna’s paper, which might be the reason makes the results to be slightly different. However, once the number of collecting terms increases, the validity of the proposed model can be expected accordingly.

**Example 2.** The second example is a magneto-elastic single layer rectangular plate made of purely piezomagnetic CoFe$_2$O$_4$ but with much longer \(L_x\). We follow the work done by Pan and Heyliger [10], the dimensions adopted are \(L_x = 1000\) m,
$L_y = 20 \text{ m}, \quad h = 1 \text{ m}, \quad S = L_y/h = 20, \quad n = 1$ and the related physical quantities are normalized as follows with all of them still remain dimensional,

$$u/S^3, \quad v/S^3, \quad w \times 100/S^4, \quad \sigma_{xx}/S^2, \quad \sigma_{yy}/S^2, \quad \sigma_{xy}/S^2$$

$$\phi/S^2, \quad \psi/S^2, \quad D_z/S, \quad \text{and} \quad B_z/S.$$

The deformation variations of the magnetostrictive plate due to external load, $\Delta P(x, y) = P_0 \sin \pi y/L_y$, applied on the top surface with magnitude $P_0 = 1 \text{ N/m}^2$ are calculated at the specified location $(x_0, y_0) = (L_x/2, S/4)$ and presented in Fig. 2.

Fig. 2(a)–(f) are the variations of the elastic displacements components $w(x, y)$ and $u_y(x_0, y_0, z)$, electric potential $\phi(x_0, y_0, z)$, magnetic potential $\psi(x_0, y_0, z)$, electric displacement $D_z$ and magnetic displacement $B_z$ along the thickness direction with boundary condition to be simply-supported around. As it can be clearly observed from Fig. 2(b) and (d), the shear deformation $u_y$ is a linear function of $z$ and the magnetic potential is a quadratic function of $z$, which have been both verified by Pan and Heyliger [10] in the case of thin-plate limit. For the normal stress components plotted in Fig. 2(g) and 2(h), a
A linear variation along the thickness direction can also be found, just as the same as which have been pointed out by Pan and Heyliger [10]. However, in the present study, the component of magnetic displacement $B_z$ is quite different from that in Pan and Heyliger's study, and presents a linear variation along the thickness direction instead of a cubic one as shown in their paper. It should be mentioned that in Pan and Heyliger's study, the boundary condition for magnetic displacement is assumed to be close-circuit, i.e. $B_z(x, y, -H/2) = B_z(x, y, H/2) = 0$, which makes the distribution becomes sinusoidal along the thickness direction.
Nevertheless, this paper proposes an open-circuit boundary condition for magnetic displacement, i.e., $\psi(x, y, -H/2) = \psi(x, y, H/2) = 0$, therefore it is reasonable for a different distribution is obtained as a result.

In this paper, the investigation on magneto-electro-elastic plate is carried out by considering the fibrous BaTiO$_3$/CoFe$_2$O$_4$ composites with respect to different volume fraction (v.f.) of BaTiO$_3$, and the corresponding material properties of this kind.

Fig. 4. Variation of (a) electric potential, (b) magnetic potential, (c) electric displacement, and (d) magnetic displacement for the fiber-reinforced MEE plate with simply-supported BCs under surface distributed load.
MEE material with 25% offset volume fractions are presented in Table 2, which is partially cited from the work conducted by Buchanan [16].

**Example 3.** In this example, a magneto-electro-elastic plate made of fiber-reinforced BaTiO$_3$/CoFe$_2$O$_4$ lamina with volume-fraction being 50% is thus presented, the dimensions, normalizations and related parameters are set to be the same with those in Example 2 except that the material constants are taken from the fifth column of Table 2. It should be noted that the electro-magneto coupling material constant $\mu_{ij}$ varies quite a lot depending upon the micromechanical theory it is used,
herein we adopt the model proposed by Li and Dunn [17]. Due to the significant change on some material constants between the pure CoFe$_2$O$_4$ and the fiber-reinforced BaTiO$_3$/CoFe$_2$O$_4$ composites, we can learn that, from Fig. 3(c)–(f), the deformation variations for a rectangular fibrous MEE thin plate due to surface load are quite different from those for a single phase case as presented in Fig. 2(c)–(f). However, despite the opposite sign appears in some quantities, the variation trend for each physical quantity along the thickness direction remains the same, i.e., quadratic relation for the potentials and linear relation for the others.

Fig. 5. (a) and (b) Deformation variation of electric potential and magnetic potential versus volume fraction for a cantilever fiber-reinforced BaTiO$_3$/CoFe$_2$O$_4$ MEE square plate subjected to unit impulse force acting on the location $(x_0, y_0)$, i.e., $\Delta P = 1.0(x - x_0, y - y_0)$. 
Fig. 6. (a) and (b) Deformation variation of electric potential and magnetic potential for the 50% fiber-reinforced BaTiO$_3$/CoFe$_2$O$_4$ MEE plate subjected to various loads with all edges simply-supported.

Fig. 7. (a) and (b) Deformation variation of electric potential and magnetic potential for the 50% fiber-reinforced BaTiO$_3$/CoFe$_2$O$_4$ MEE plate subjected to various loads with all edges clamped.

Starting from this point, the fibrous BaTiO$_3$/CoFe$_2$O$_4$ composites with respect to various volume-fractions working as a magneto-electro-elastic plate will be discussed based on the Kirchoff hypothesis and the thin-plate theory. The simplified
The governing equation, Equation (38), will be examined subjected to various kinds of surface applied load, and the deformation behavior with different boundary conditions imposed on the plate will also be inspected.

The dimensions of the MEE plate is set to be $L_x \times L_y \times H = 1\text{ m} \times 1\text{ m} \times 0.05\text{ m}$ unless otherwise mentioned, however, any kind of dimensions can be applied as long as the span-to-thickness ratio is satisfying the requirement for thin-plate theory, i.e., $L_x/H > 10$. The plate surfaces are assumed to be traction free except on the top or bottom surface, on which a $z$-direction surface load is applied. The external load can be of any type possibly occurs in the study of MEE plate, however, in order to observe the variation of the deformations, three commonly seen static forces are performed in this paper. They are

1. (1) uniform load, i.e., $P(x, y) = P_0$,
2. (2) distributed load, i.e., $P(x, y) = P_0 X_M(x) Y_N(y)$ and
3. (3) concentrated load, i.e., $P(x, y) = P_0 (x - x_0, y - y_0)$.

The geometric boundary conditions imposed on the MEE plate can be any one of the combinations for free, clamped and simply-supported edges; however, in this study, only the boundary conditions of simply-supported around, clamped around and cantilever plates are conducted as numerical examples. It should be noted that the deformation responses for the MEE plate under static loads will be calculated at a fixed horizontal coordinate $(x_0, y_0)$, however, due to the different plate characteristic with respect to different boundary conditions, the horizontal coordinates will be various according to the corresponding boundary conditions. That is, for SSSS plate location is chosen to be at $(x_0, y_0) = (0.5 L_x, 0.5 L_y)$, for CCCC plate at $(x_0, y_0) = (0.5 L_x, 0.5 L_y)$ and for CFFF plate at $(x_0, y_0) = (L_x, 0.5 L_y)$.

Fig. 4(a)–(d) are the deformation variation of electric potential and magnetic potential for the fiber-reinforced BaTiO$_3$/CoFe$_2$O$_4$ MEE plate with simply-supported edges and surface distributed load $\Delta P = \sin(\pi x/L_x) \sin(\pi y/L_y)$ with respect to various volume fraction of BaTiO$_3$. It can be found that the largest magnitude of electric potential is induced in the fiber-reinforced MEE lamina with 75% v.f. of BaTiO$_3$, which is even bigger than the second one induced in the pure piezoelectric BaTiO$_3$ plate (i.e., 100% v.f.) followed by the 50% case and 25% case. This incident is indeed interesting and can be of great value for someone devoted to the design of MEE structures. However, this incident is happened mainly due to the dramatic change of the material parameters adopted in the present study, which is proposed by Ref. [17]. As we can seen from the expression for electric potential, say Eq. (53), the magnitude is actually determined by $\Delta U$, where $\Delta = \epsilon_{33} H_{33} - d_{33}^2$ and $A_1 = \epsilon_{31} H_{33} - d_{33}^2 H_{31}$. It can be detected that the material constants stated in Table 1 reveal huge variation with respect to the volume fraction; nevertheless, the 75% lamina happened to be the one with the largest quotient as a result. On the part of magnetic potential, it is obvious that the bi-material composite gets more magnetic effect...
than the single-phase constituent, i.e., 50% of the composite is the biggest one, 75% is closely tailed and 25% follows, for the pure BaTiO3 and pure CoFe2O4 plates, zero and tiny magnitudes are indicated.

For the fibrous composite presented in Fig. 4(c) and (d), the electric and magnetic displacements, also called fluxes, can be found to be linearly dependent on the z variable and the slope varies slightly with respect to the volume fraction of the MEE plate. However, if we near watch the magnitudes, we can find that all of them are pretty slim. This is due to the nature of the material parameter itself, also because of the close-circuit restriction we impose on the model. Nevertheless, if the electric and magnetic boundary conditions are chosen otherwise, the magnitudes for electric and magnetic displacements may be changed in a different way. In the present study, only the close-circuit MEE thin plate is considered, therefore the author hereafter leave out all the presentations and discussions for the electric displacement and the magnetic displacement due to their insignificant effects.

Since the deformation variations for the fibrous MEE plates with clamped-around edges and under uniform applied load \( \Delta P=1 \text{ N/m}^2 \) are quite similar to those behaviors presented in Fig. 4, the author therefore skip this case and directly go to the case for the cantilever fibrous MEE plate. Also the deformation variations for the electric and magnetic displacements are neglected due to their small magnitudes both approaching to zero.

Fig. 5(a) and (b) are the deformation variation of electric potential, magnetic potential versus volume fraction for a cantilever fiber-reinforced BaTiO3/CoFe2O4 MEE plate subjected to unit impulse force acting on the location \((x_0, y_0)\), i.e., \( \Delta P = 1 \delta(x - x_0, y - y_0) \). The deformation behavior of a cantilever MEE plate is quite similar to the corresponding one of a simply-supported MEE plate except for the concavity. Owing to the different characteristics of MEE plate with different boundary conditions and subjected to different types of applied load, the sign change on the concavity is reasonable and expected.

The deformation variation of electric potential and magnetic potential for the 50% fiber-reinforced BaTiO3/CoFe2O4 MEE plate subjected to various loads with all edges simply-supported are demonstrated in Fig. 6(a) and (b). And the corresponding deformation variations for the fibrous MEE composite with all edges clamped are depicted in Fig. 7(a) and (b). Followed by Fig. 8(a) and (b), the same illustrations for the fibrous MEE cantilever plate are provided. As we can see from these figures, the concentrated applied load always stimulate much stronger deformation as we expected and followed by the distributed applied load, uniform applied load seems to be less operative.

4. Conclusions

A closed form expressions for the bending problem of a magneto-electro-elastic (MEE) rectangular thin composite are derived based on the Kirchhoff hypothesis and classical thin-plate theory. The governing equation in terms of the transverse displacement only is presented in a rather compact form, and the elastic displacements, electric potential and magnetic induction for a magneto-electro-elastic (MEE) multilayer plate are implemented analytically.

It has been shown that the material coefficients for the MEE constituent vary a lot according to the way it is fabricated as well as the volume-fraction of BaTiO3 it contains. The deformation variations for the MEE thin plate with closed-circuit electric restriction are evaluated with respect to various boundary conditions, and the effects of the volume-fractions are investigated in detail. It can be found that the shear deformation is linearly dependent on the transverse deformation, whereas the electric and magnetic potential are both of quadratic variation along the thickness direction. In particular, the deformation behavior for a single phase material can be found to be quite different from the multilayer one in either the magnitude or the sign it is induced.

Due to the dramatic change on some material constants between the single-phase continuum and the fibrous MEE composites, it should be noted that the deformation variations for a fibrous MEE thin plate induced by the applied load are actually distinct from those for a pure material case. When a multilayer MEE thin plate is under consideration, the volume fraction of BaTiO3 in the composite indeed plays an important role on the structure characteristics. However, the effect of volume fraction seems to be varying from case to case, and the interactively coupled quantities for a multilayer thin plate differ quite irregularly with respect to the volume fraction.

The present study provides some commonly seen examples for the magneto-electro-elastic (MEE) rectangular thin plate under the action of three kinds applied loads, and offers the discrepancy between the single-phase continuum and the fibrous MEE composites on the deformation variation of electric potential and magnetic induction with respect to various typical boundary conditions. This work proposed a much easier and systematic way to seek for the analytic solutions for the deformation characteristics of a multilayer MEE thin plate, and should be of interest in the practice of structure design with the medium to be fully coupled.

Acknowledgements

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References


